Fixed-Order Dynamic Compensation for Multivariable Linear Systems

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This paper considers the design of fixed-order dynamic compensators for multivariable time-invariant linear systems, minimizing a linear quadratic performance cost functional. Attention is given to robustness issues in terms of multivariable frequency domain specifications. An output feedback formulation is adopted by suitably augmenting the system description to include the compensator states. Either a controller or observer canonical form is imposed on the compensator description to reduce the number of free parameters to its minimal number. The internal structure of the compensator is prespecified by assigning a set of ascending feedback invariant indices, thus forming a Brunovsky structure for the nominal compensator.

Nomenclature

A,B,C	= system matrices
$\tilde{A},\tilde{B},\tilde{C}$	= augmented system matrices
E	= expected value
G	= output feedback matrix
$\tilde{G}$	= augmented output feedback matrix
H^0	= prespecified compensator output matrix
I	= identity matrix
J	= cost function
k_i	= predefined controllability indices of P
l_i	= predefined observability indices of P
n,m,p	= system state, input, and output dimensions
nc	= compensator state dimension
nm,np	= input and output state shaping filter dimensions
N^0	= prespecified compensator input matrix
P,N,H	= compensator matrices
P_z	= compensator feedback gain matrix
P^0	= prespecified compensator matrix in Brunovsky form
Q,R	= state and control weighting matrices
$Q_y(j\omega)$	= hermitian rational output weighting matrix
$R(j\omega)$	= hermitian rational input weighting matrix
s	= Laplace transform variable
u_c	= input from compensator
u_1	= new input
w_y	= output shaping filter state
w_u	= input shaping filter state
x,u,y	= system state, input and output
$\tilde{x},\tilde{u},\tilde{y}$	= augmented system state, input and output
y_1	= new output
z	= compensator state
ω	= frequency
ρ	= scalar control weighting

Subscripts

u	= input shaping filter matrices
y	= output shaping filter matrices

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Introduction

It is well known that linear quadratic regulator (LQR) synthesis methods have guaranteed stability margins. Unfortunately, these properties hold only in the case of full state feedback. Observer-based compensator design techniques exist to estimate the unavailable plant states, making the LQR design viable. However, this combination of state estimation and regulation may result in a compensator design with poor stability margins, even though the separate designs are robust. It has been shown that an LQR design for minimum-phase plants can be recovered via an asymptotic method called loop transfer recovery (LTR).[1] The LTR method relies on a cheap control formulation with a subset of the compensator dynamics becoming fast. In this postanalog era, finite word length, sampling rate, or time delay may impede the use of high-gain controllers when implemented digitally. Several other shortcomings of the LTR method that are related to plant inversion are discussed in Ref. 2. Aside from the robustness issue, the order of the compensator when designed for large scale systems may prove unwarranted.

Optimal output feedback design of fixed-order compensators introduced in the early 1970's[3] has received limited attention over the years. Part of the difficulty in synthesizing optimal dynamic compensators is that the standard approach of adjoining the compensator dynamics to the plant dynamics (and reformulating as a static gain output feedback design) results in an overparameterized formulation. This is a direct consequence of the fact that the compensator lacks a predefined structure, which invariably results in difficulties with convergence to an optimal solution. The uniqueness of the compensator parameters is achieved by using a minimal realization of the compensator and requires creating a set of constraints among the compensator elements.[4,5] Another difficulty rests with the inability to characterize the stability margin properties. Unlike the algebraic Ricatti equation that arises in the case of full state feedback, the necessary conditions that result from the optimal output feedback problem are not conducive to analysis in the frequency domain.

In this paper, we adopt a canonical form for the compensator that minimizes the number of free parameters. Two types of control structures are formulated: the controller form and the observer form. Both forms require a prespecification of the compensator's initial structure. Two salient features arise from adopting these canonical forms: 1) a compensator design without direct output feedthrough can be readily implemented via an optimal output feedback routine without the need to zero unwanted feedback elements; and 2) this structure of the compensator does not require penalizing the compensator states in order to guarantee a nonzero or coupled solution, a problem that occurs when following a standard output feedback design.

Problems can arise when trying to redesign the compensator after examining the stability margin properties for the feedback loops broken at the input or output of the plant. The approach taken in this paper is to extend the method of frequency-dependent cost functionals[6] to the output feedback problem. Frequency-shaped measurement and control weightings are used to expand the compensator so as to enhance the system's closed-loop robustness. A natural extension is to include the robust regulator problem, where an internal model for external disturbances and input commands is added to the compensator.[7]

Dynamic Output Feedback

Consider the multiloop feedback control system for the plant described by

$$\dot{x} = Ax + Bu \quad x \in R^n, u \in R^m \tag{1}$$

$$y = Cx \quad y \in R^p \tag{2}$$

and controlled by a dynamic compensator of order n_c

$$u = -Hz \tag{3}$$

$$\dot{z} = -Pz - Ny \quad z \in R^{nc} \tag{4}$$

The order of the compensator nc is chosen a priori subject to the stabilizability to the closed-loop system. A lower bound on the compensator size required to guarantee stability of the nominal closed-loop system is presently unavailable and will not be addressed in this paper.

The plant described in Eqs. (1) and (2) can be characterized by the rational matrix transfer function description,

$$G(s) = C(sI - A)^{-1}B \tag{5}$$

Similarly, the compensator described in Eqs. (3) and (4) can be characterized as

$$K(s) = H(sI + P)^{-1}N \tag{6}$$

The strict properness of the compensator $K(s)$ avoids the direct passage of unmodeled high-frequency measurement noise not accounted for, but implicitly assumed, in the design synthesis.

Defining $\tilde{x}' = [x', z']$, $\tilde{y}' = [y', z']$ and $\tilde{u}' = [u', u_c']$, $u_c = \dot{z}$, the dynamic compensator design can then be expressed in terms of a standard output feedback problem as

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u}, \quad \tilde{x}' = [x', z'] \quad \tilde{x} \in R^{n+nc} \tag{7}$$

$$\tilde{y} = \tilde{C}\tilde{x} \quad \tilde{y} \in R^{p+nc} \tag{8}$$

$$\tilde{u} = -\tilde{G}\tilde{y} \quad \tilde{u} \in R^{m+n_c} \tag{9}$$

where

$$\tilde{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B & 0 \\ 0 & I_{nc} \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} C & 0 \\ 0 & I_{nc} \end{bmatrix} \quad \tilde{G} = \begin{bmatrix} 0 & H \\ N & P \end{bmatrix} \tag{10}$$

The output feedback problem is then optimized according to the following augmented performance index:

$$J(P,N,H) = E_{\tilde{x}_0}\left\{\int_0^\infty [\tilde{x}'Q\tilde{x} + \tilde{u}'R\tilde{u}]\,\mathrm{d}t\right\} \tag{11}$$

where the expectation is taken over some initial distribution on $\tilde{x}_0$.

Since the input/output characteristics of the compensator are invariant with respect to any nonsingular transformation T, the compensator representation is in at least one sense overparameterized. When the compensator parameters (P,N,H) are sought in conjunction with an optimal LQ output feedback formulation, this redundancy in the compensator parameters generally leads to convergence difficulties.

A second computational problem in the above formulation is that the upper left-hand block of the output feedback gain matrix $\tilde{G}$ must be constrained to zero. For large-order controller designs, introducing a constraint of $\tilde{G}$ slows convergence considerably.

Given the triple (P,N,H), there exist various similarity transformations that reduce the number of free parameters. In Ref. 4, P is arranged in a block diagonal form with each block in companion form of degree two. Unfortunately, this formulation requires constraining many elements of $\tilde{G}$ to zero. Hence, an important aspect of this section is the construction of a compensator description with a minimal number of free parameters and with an output feedback formulation wherein $\tilde{G}$ has no zero elements.

It is known that, for a controllable pair (P,N), there exists a set of controllability indices which are invariant with respect to any basis change. Likewise, for an observable pair (P,H), there exists a set of observability indices that share this same invariance property. Given either a controllable or observable system and their respective controllability or observability indices, the maximum number of parameteres needed to uniquely define the input/output description can be reduced from $nc(nc + mc + p)$ to $nc(m + p)$.[8] Two canonical forms of the compensator triple (P,N,H) are used here as a basis for fixed-order compensator synthesis, namely, the controller form and the observer form.

Controller Canonical Form

The following theorem depicts a structure for the compensation by providing a controller canonical form.[9]

Theorem 1: For a controllable compensator [Eqs. (3) and (4)] with N full rank, the triple (P,N,H) can be transformed into a controller canonical form, redefined as (P,N^0,H), where

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1p} \\ P_{21} & P_{22} & \cdots & P_{2p} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ P_{p1} & P_{p2} & \cdots & P_{pp} \end{bmatrix}$$

$$P_{ii} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & & 1 \\ x & x & x & \cdots & x \end{bmatrix}_{k_i \times k_i},$$

$$\underset{i \neq j}{P_{ij}} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ x & x & x & \cdots & x \end{bmatrix}_{k_i \times k_i}$$

x = possible nonzero element
N^0 = block diag$\{[0 \cdots 0\ 1]^t_{k_i x 1}, i = 1, \ldots p\}$
H = arbitrary

if the controllability indices k_i of the original triple (P,N,H) satisfy the following conditions:

1) $\sum_{i=1}^{p} k_i = nc$.

2) $k_i \leq k_i + 1$.

If the indices are not in ascending order, one can rearrange the columns of N^0 such that condition 2 holds.

The significance of the above theorem is that one can a priori choose the internal structure of the compensator by assigning a controllability index for each plant output in accordance with condition 1, reorder the elements of y to satisfy condition 2, and then optimize the $nc(m+p)$ compensator parameters. This leads to the following controller canonical form for the compensator:

$$u = -Hz, \quad u \in R^m \tag{12}$$

$$\dot{z} = P^0 z + N^0 u_c - N^0 y, \quad z \in R^{nc} \tag{13}$$

$$u_c = -P_z z, \quad u_c \in R^p \tag{14}$$

where

$$P^0 = \text{block diag}[P_1^0, \ldots, P_p^0]$$

with

$$P_i^0 = \begin{bmatrix} 0 & 1 & 0 & \ldots & 1 \\ 0 & & 0 & 1 & 0 \\ \cdot & & \cdot & \cdot & \cdot \\ 0 & & 0 & 0 & 1 \\ 0 & & 0 & 0 & \ldots 0 \end{bmatrix}_{k_i \times k_i} \tag{15}$$

Matrices H and P_z are free-parameter matrices with dimensions $(m \times nc)$ and $(p \times nc)$, respectively.

Now, augment the plant states to include the compensator states, that is, let $\tilde{x}^t = [x^t, z^t]$, $\tilde{y} = z$, and $\tilde{u}^t = [u^t, u_c^t]$. The extended output feedback system is then given by

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u} \tag{16}$$

$$\tilde{y} = \tilde{C}\tilde{x} \tag{17}$$

$$\tilde{u} = -\tilde{G}\tilde{y} \tag{18}$$

where

$$\tilde{A} = \begin{bmatrix} A & 0 \\ -N^0 C & P^0 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B & 0 \\ 0 & N^0 \end{bmatrix}$$

$$\tilde{C} = [0 \quad I_{nc}], \quad \tilde{G} = \begin{bmatrix} H \\ P_z \end{bmatrix} \tag{19}$$

Unlike the standard output feedback formulation [Eqs. (7–10)], the above formulation minimizes the number of free parameters and has no zero elements in $\tilde{G}$. The ith row of P_z in Eq. (19) corresponds to the bottom row of free parameters in a P_{ij}.

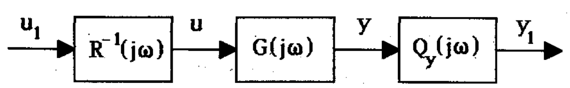

Fig. 1 Relationship between plant and shaping filters.

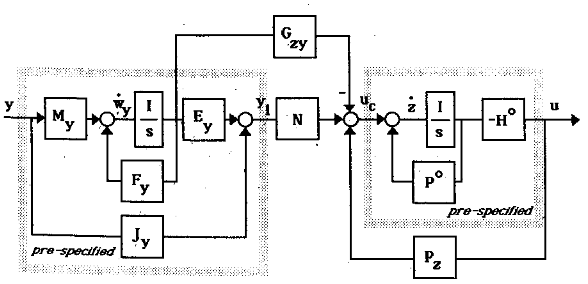

Fig. 2 Compensator in observer form with output frequency shaping.

Observer Canonical Form

Next, an observer canonical form for the compensator is presented. The next theorem characterizes the structure of this compensator.

Theorem 2: For an observable compensator [Eqs. (3) and (4)] with H full rank, the triple (P,N,H) can be transformed into an observer canonical form, redefined as (P,N,H^0), where

$$P = \begin{bmatrix} P_{11} & P_{12} & \ldots & P_{1m} \\ P_{21} & P_{22} & \ldots & P_{2m} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ P_{m1} & P_{m2} & \ldots & P_{mm} \end{bmatrix}$$

$$P_{ii} = \begin{bmatrix} 0 & 0 & \ldots & 0 & x \\ 1 & 0 & & 0 & x \\ 0 & 1 & & 0 & x \\ \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \ldots & 0 & x \end{bmatrix}_{\ell_i \times \ell_i}$$

$$\underset{i \neq j}{P_{ii}} = \begin{bmatrix} 0 & 0 & \ldots & 0 & x \\ 0 & 0 & & 0 & x \\ 0 & 0 & & 0 & x \\ \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \ldots & 0 & x \end{bmatrix}_{\ell_i \times \ell_i}$$

x = possible nonzero element
$H^0 = \text{block diag}\{[0 \ldots 0 \quad 1]_{1 \times \ell_i}, i = 1, \ldots m\}$
N = arbitrary
if the observability indices ℓ_i of the original triple (P,N,H) satisfy the following conditions:

1) $\sum_{i=1}^{m} \ell_i = nc$.

2) $\ell_i \leq \ell_{i+1}$.

Just as in the controller form, if the observability indices are not in ascending order, one can arrange the rows of H^0 such that condition 2 holds.

From theorem 2, the compensator equations (3) and (4) can now be written as

$$u = -H^0 z, \quad u \in R^m \tag{20}$$

$$\dot{z} = P^0 z + u_c, \quad z \in R^{nc} \tag{21}$$

$$u_c = P_z u - Ny, \quad u_c \in R^{nc} \tag{22}$$

where

$$P^0 = \text{block diag}[P^0{}_1, \ldots, P_m^0]$$

with

$$P_i^0 = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \ldots & 0 & 0 \end{bmatrix}_{\ell_i \times \ell_i} \tag{23}$$

Now N and P_z are free parameter matrices with dimensions $(nc \times p)$ and $(nc \times m)$, respectively. Here the ith column of P_z corresponds to the last column of free parameters in P_{ij}.

The compensator design is now formulated as an output feedback problem for the observer form. Let $\tilde{x}' = [x', z']$, $\tilde{u} = u_c$, and $\tilde{y}' = [y', -u']$. The augmented system of Eqs. (16–18) in conjunction with the system matrices

$$\tilde{A} = \begin{bmatrix} A & -BH^0 \\ 0 & P^0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ I_{nc} \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} C & 0 \\ 0 & H^0 \end{bmatrix}, \quad \tilde{G} = [N \quad P_z] \tag{24}$$

defines the output feedback problem. Just as in the controller form, the above formulation minimizes the number of free parameters and has no zero elements in $\tilde{G}$.

In comparing the controller and observer forms, it is interesting to note that their minimum dimensions are p and m, respectively. This corresponds to the case where $k_i = \ell_i = 1$. Thus, the observer form may be preferred when $p > m$.

Frequency-Shaped Dynamic Compensation

Consider the linear quadratic cost functional in the frequency domain

$$J = E\left\{\frac{1}{2}\pi \int_0^\infty [\tilde{x}^*(j\omega)Q\tilde{x}(j\omega) + y^*(j\omega)\bar{Q}_y(j\omega)y(j\omega) + u^*(j\omega)\bar{R}(j\omega)u(j\omega) + u_c^*(j\omega)R_c u_c(j\omega)]\,\mathrm{d}\omega\right\} \tag{25}$$

where Q and R_c are constant coefficient matrices. The hermitian rational matrices $\bar{Q}_y(j\omega)$ and $\bar{R}(j\omega)$ are non-negative and positive definite, respectively, and are a function of squared frequency ω^2. They are defined as

$$\bar{Q}_y(j\omega) = Q_y^*(j\omega)Q_y(j\omega) \tag{26}$$

and

$$\bar{R}(j\omega) = R^*(j\omega)R(j\omega) \tag{27}$$

where $(\ )^*$ denotes a complex conjugate transpose. Furthermore, the rational matrices $Q_y(j\omega)$ and $R(j\omega)$ represent shaping filters and define a new input y_1 and a new output u_1, as

$$y_1(j\omega) = Q_y(j\omega)y(j\omega) \tag{28}$$

and

$$u_1(j\omega) = R(j\omega)u(j\omega) \tag{29}$$

The relationship between the new output y_1 and new input u_1 and the plant output y and input u is illustrated in Fig. 1.

The frequency-dependent cost functional can be expressed in the time domain by substituting the new output and input states for the frequency dependent weighting matrices in Eq. (25) and then applying Parseval's theorem to obtain

$$J = E\left\{\int_0^\infty [\tilde{x}'Q\tilde{x} + y_1'y_1 + u_1'u_1 + u_c'R_c u_c]\,\mathrm{d}t\right\} \tag{30}$$

The LQ optimization can now be viewed as an output feedback design with the new ouput y_1 and the new input u_1 and the proper matrices $Q_y(s)$ and $R^{-1}(s)$ being realized as

$$Q_y(s) = E_y(sI - F_y)^{-1}M_y + J_y \tag{31}$$

or

$$\dot{w}_y = F_y w_y + M_y y, \quad w_y \in R^{np} \tag{32}$$

$$y_1 = E_y w_y + J_y y \tag{33}$$

and

$$R^{-1}(s) = E_u(sI - R_u)^{-1}M_u + J_u \tag{34}$$

or

$$\dot{w}_u = F_u w_u + M_u u_1, \quad w_u \in R^{nm} \tag{35}$$

$$u = E_u w_u + J_u u_1 \tag{36}$$

To illustrate the use of frequency shaping, the dynamic compensator in observer canonical form will be considered below. A parallel procedure exists for the controller canonical form.[7] Expanding the compensator form of Eqs. (20–22) to include the input and output filter states, we have

$$u_i = -H^0 z \tag{37}$$

$$\dot{z} = P^0 z + u_c \tag{38}$$

$$u_c = -Ny_1 - G_{zy}w_y - G_{zu}w_u + P_2 u_1 \tag{39}$$

Reformulation as a static output feedback problem using the augmented system description [Eqs. (16–18)] where $\tilde{x}' = [x', w_y', w_u', z']$, $\tilde{y}' = [y_1', w_y', w_u', -u_1']$, and $\tilde{u} = u_c$ results in

$$\tilde{A} = \begin{bmatrix} A & 0 & BE_u & -BJ_uH^0 \\ M_yC & F_y & 0 & 0 \\ 0 & 0 & F_u & -M_uH^0 \\ 0 & 0 & 0 & P_0 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_{nc} \end{bmatrix}$$

$$\tilde{C} = \begin{bmatrix} J_yC & E_y & 0 & 0 \\ 0 & I_{np} & 0 & 0 \\ 0 & 0 & I_{nm} & 0 \\ 0 & 0 & 0 & H^0 \end{bmatrix}$$

$$\tilde{G} = [N \quad G_{zy} \quad G_{zu} \quad P_z] \tag{40}$$

The composite compensator with output frequency shaping is shown in Fig. 2 and the composite compensator with input frequency shaping is shown in Fig. 3. Further extensions to the robust regulator problem for disturbance rejection and command following using the internal model principle can be found in Ref. 7.

Application

This section illustrates a robust fixed-order compensator design for a two-output/single-input system. The example, a design for an azimuth pointing control system for a multiple-mirror telescope, is taken from Ref. 10 where a frequency-shaped filter was used to improve the robustness of an observer-based compensator. The numerical method described in Ref. 11 was used to obtain the output feedback gain matrix.

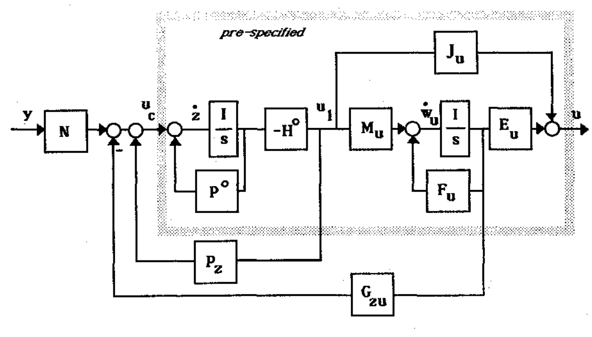

Fig. 3 Compensator in observer form with input frequency shaping.

The plant is described by the triple (A,B,C) as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1.4\times 10^4 & -0.833 & 1.14\times 10^6 & 0 \\ 0 & 0 & 0 & 1 \\ 6.03 & 0 & 603.0 & -1.47\times 10^{-3} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0.278 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The state and control vectors are

$$x = \begin{bmatrix} \theta_M \\ \omega_M \\ \theta_T \\ \omega_T \end{bmatrix} \begin{matrix} \text{motor shaft angle, rad} \\ \text{motor shaft rate, rad/s} \\ \text{azimuth pointing angle, rad} \\ \text{telescope angle rate, rad/s} \end{matrix}$$

$u = t_c$ commanded motor torque, 1lb · ft.

The measurements are ω_M and θ_T. For design purposes, the plant triple (A,B,C) is transformed to modal coordinates as

$$A = \begin{bmatrix} -0.396 & 109.6 & 0 & 0 \\ -109.6 & -0.396 & 0 & 0 \\ 0 & 0 & -0.0432 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.264 \\ 0.316 \\ 0.316 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.015 & 0.999 & -0.0432 & 0 \\ 0 & 0 & -0.0999 & 0.01 \end{bmatrix} \tag{41}$$

In Ref. 10, an observer-based compensator was designed based on the low-frequency modes (0 and -0.0432) by truncating the transformed plant model. The resulting compensator was then cascaded with the full-order system.

Here, the full-order model is used. In this study, a first-order compensator in observer form is used to stabilize the system. This form permits feeding back two measurements without requiring a second-order compensator. For this example, Eqs. (20–22) reduce to

$$u = -z, \quad \dot{z} = u_c$$

$$u_c = -n_1\omega_M - n_2\theta_T + p_z u \tag{42}$$

The performance index for the augmented system was chosen as

$$J = E\left\{\int_0^\infty [\tilde{x}'Q\tilde{x} + \rho_c u_c^2]\,\mathrm{d}t\right\} \tag{43}$$

where $\tilde{x}' = [x',z]$ and

$$Q = \mathrm{diag}[0,0,Q_t,0,]; \quad Q_t = \begin{bmatrix} 10^7 & -10^7 \\ -10^7 & 10^7 \end{bmatrix}$$

$$\rho_c = 1$$

$$E\{\tilde{x}_0\tilde{x}_0'\} = I_5 \tag{44}$$

The resulting compensator gains are

$$\tilde{G} = [n_1, n_2, p_z]$$
$$= [1.09\times 10^4, 1.97\times 10^6, 4.88] \tag{45}$$

The open-loop transfer function for the loop broken at the input is listed in Table 1 and the frequency response is shown in Fig. 4. There exists only 3 dB of gain margin.

A notch filter is used to improve the stability margin for the above design by defining

$$R^{-1}(s) = (s^2 + 2\zeta\omega s + \omega^2)/(s^2 + 2\zeta_d\omega s + \omega^2) \tag{46}$$

where ω is the resonant frequency of the flexible mode, ζ the damping ratio, and ζ_d the desired damping ratio. From Eq. (41), $\omega = 110$ rad/s, $\zeta = 0.005$, and ζ_d was selected as 0.707.

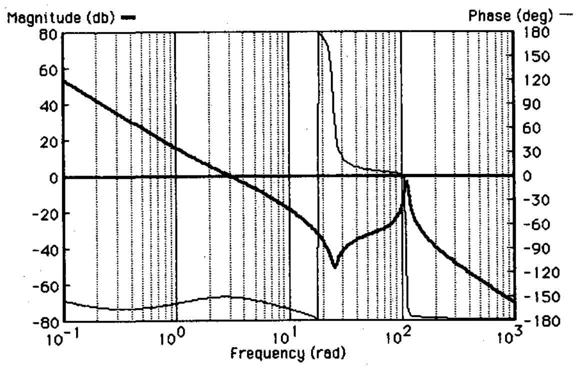

Fig. 4 Frequency response of $u(s)/u'(s)$ for the first-order compensator.

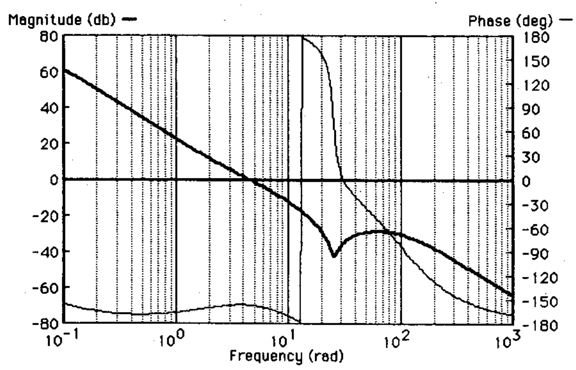

Fig. 5 Frequency response of $u(s)/u'(s)$ for the third-order compensator.

Table 1 Loop transfer functions for loop broken at input

First-order compensator	
	$\dfrac{u(s)}{u'(s)} = \dfrac{303.8\,(s+1.72)[(s-1.346)^2+(24.81)^2]}{s(s+0.0432)(s+4.881)[(s+0.396)^2+(109.6)^2]}$
Third-order compensator with frequency shaping	
	$\dfrac{u(s)}{u'(s)} = \dfrac{594.1\,(s+2.921)[(s-1.912)^2+(25.33)^2]\,[(s+0.56)^2+(110)^2]}{s(s+0.0432)(s+8.0)[(s+88.35)^2+(52.6)^2][(s+0.396)^2+(109.6)^2]}$

The notch filter is realized as

$$\dot{w}_1 = w_2$$
$$\dot{w}_2 = -\omega^2 w_1 - 2\zeta_d \omega w_2 + u_1$$
$$u = 2(\zeta - \zeta_d)\omega w_2 + u_1 \tag{47}$$

and the compensation is given by

$$u = -z, \quad \dot{z} = u_c$$
$$u_c = -n_1\omega_M - n_2\theta_T - g_1 w_1 - g_2 w_2 + p_z u \tag{48}$$

The performance index is given by Eq. (43) for the augmented system with $\tilde{x} = [x^t, w_1, w_2, z]$ and

$$Q = \mathrm{diag}[0,0,Q_t,0,0,0]; \quad Q_t = \begin{bmatrix} 10^7 & -10^7 \\ -10^7 & 10^7 \end{bmatrix}$$
$$\rho_c = 0.001$$
$$E\{\tilde{x}_0 \tilde{x}_0^t\} = I_7 \tag{49}$$

The resulting compensator gains were

$$\tilde{G} = [n_1, n_2, g_1, g_2, p_z]$$
$$= [-2.13\times10^4, \ -6.81\times10^6, 2.68\times10^4, 4.65\times10^4, 29.17] \tag{50}$$

The loop transfer function for the loop broken at the input is listed in Table 1 and the frequency response of the compensator with frequency shaping is shown in Fig. 5. The gain margin is increased tenfold to 30 dB and the phase margin is 28 deg. These results compare favorably with the observer-based compensator design. Here, the final compensator is third-order, while in Ref. 10, the observer-based compensator with frequency shaping on ω_M, is fourth-order. More importantly, this design was based on a higher-order model. If the four-state model was used in Ref. 10, the resulting compensator would have been sixth-order by that design approach.

Summary

A method has been presented for linear quadratic synthesis of fixed-order dynamic compensators without direct output feedback. Controller and observer canonical structures are formulated, which minimize the number of free parameters to be optimized and which do not require constraining parts of the gain matrix to zero. Both compensator structures permit the use of frequency-dependent cost functionals, which can be used to improve the robustness of a fixed-order compensator design. Frequency-shaped measurement and/or control weighting can be employed.

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